Symbolab

Solution

Range of $\sqrt{x^2-6x+8}$: $\left[\begin{array}{ll}\text{Solution:} & f(x)\ge 0\\ \text{Interval Notation:} & [0,\infty)\end{array}\right]$

Steps

Function range definition

The set of values of the dependent variable for which a function is defined

Find the minimum and maximum value in each defined interval and unite the results

Hide Steps

Domain of $\sqrt{x^2-6x+8}$: $x\le 2$ or $x\ge 4$

Domain definition

The domain of a function is the set of input or argument values for which the function is real and defined

Find non – negative values for radicals: $x\le 2$ or $x\ge 4$ Show Steps

The function domain

$x\le 2$ or $x\ge 4$

Hide Steps

Extreme Points of $\sqrt{x^2-6x+8}$: Minimum$(2,0)$, Minimum$(4,0)$

First Derivative Test definition

Suppose that $x=c$ is a critical point of $f(x)$ then,
If $f'(x)>0$ to the left of $x=c$ and $f'(x)<0$ to the right of $x=c$ then $x=c$ is a local maximum.
If $f'(x)<0$ to the left of $x=c$ and $f'(x)>0$ to the right of $x=c$ then $x=c$ is a local minimum.
If $f'(x)$ is the same sign on both sides of $x=c$ then $x=c$ is neither a local maximum nor a local minimum.

Find the critical points: $x=2, x=4$ Show Steps

Domain of $\sqrt{x^2-6x+8}$: $x\le 2$ or $x\ge 4$ Show Steps

Combine the critical point(s): $x=2, x=4$ with the domain

The function monotone intervals are:

$-\infty<x<2, 4<x<\infty$

Check the sign of $f'(x)=\frac{x-3}{\sqrt{x^2-6x+8}}$ at each monotone interval

Check the sign of $\frac{x-3}{\sqrt{x^2-6x+8}}$ at $-\infty<x<2$: Negative Show Steps

Check the sign of $\frac{x-3}{\sqrt{x^2-6x+8}}$ at $4<x<\infty$: Positive Show Steps

Use the First Derivative Test Variation for discontinous functions Show Steps

Summary of the monotone intervals behavior

	$-\infty<x<2$	$x=2$	$x=4$	$4<x<\infty$
Sign	$-$	NA	NA	$+$
Behavior	Decreasing	Minimum	Minimum	Increasing

Plug the extreme point $x=2$ into $\sqrt{x^2-6x+8}$ $\Rightarrow$ $y=0$

Minimum$(2,0)$

Plug the extreme point $x=4$ into $\sqrt{x^2-6x+8}$ $\Rightarrow$ $y=0$

Minimum$(4,0)$

Minimum$(2, 0)$, Minimum$(4, 0)$

Find the range for the interval $-\infty < x \le 2$: $0 \le f(x) < \infty$

Hide Steps

Compute the values of the function at the edges of the interval:

For $x = 2$, the function value is $f(2) = 0$

$\lim_{x\to -\infty}\left(\sqrt{x^2-6x+8}\right) = \infty$

Show Steps

The interval has a minimum point at $x = 2$ with value $f(2) = 0$

Combine the function value at the edge with the extreme points of the function in the interval:

Minimum function value at the domain interval $-\infty < x \le 2$ is 0

Maximum function value at the domain interval $-\infty < x \le 2$ is ∞

Therefore the range of $\sqrt{x^2-6x+8}$ at the domain interval $-\infty < x \le 2$ is

$0 \le f(x) < \infty$

Find the range for the interval $4 \le x < \infty$: $0 \le f(x) < \infty$

Hide Steps

Compute the values of the function at the edges of the interval:

For $x = 4$, the function value is $f(4) = 0$

$\lim_{x\to\infty}\left(\sqrt{x^2-6x+8}\right) = \infty$

Show Steps

The interval has a minimum point at $x = 4$ with value $f(4) = 0$

Combine the function value at the edge with the extreme points of the function in the interval:

Minimum function value at the domain interval $4 \le x < \infty$ is 0

Maximum function value at the domain interval $4 \le x < \infty$ is ∞

Therefore the range of $\sqrt{x^2-6x+8}$ at the domain interval $4 \le x < \infty$ is

$0 \le f(x) < \infty$

Combine the ranges of all domain intervals to obtain the function range

$f(x) \ge 0$ or $f(x) \ge 0$

$f(x) \ge 0$

Graph

Plotting: $y = \sqrt{x^2 - 6x + 8}$